UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2015/2016 Academic Session

June 2016

MAA101 – Calculus for Science Students I
[Kalkulus untuk Pelajar Sains I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed material before you begin the examination.

[*Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*]

Instructions: Answer **SEVEN** (7) questions.

*[**Arahan:** Jawab **TUJUH** (7) soalan].*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. Find the following limits, if they exist.

(a) $\lim\limits_{x\to 0}\left(\dfrac{\sqrt{x+1}-1}{x}\right)$

(b) $\lim\limits_{x\to 2}\dfrac{8x-16}{x^2-4}$

(c) $\lim\limits_{x\to 0}\dfrac{2x^2+7}{\sqrt{3x^4+4}}$

[8 marks]

1. Dapatkan had berikut, jika ianya wujud.

(a) $\underset{x\to 0}{had}\left(\dfrac{\sqrt{x+1}-1}{x}\right)$

(b) $\underset{x\to 2}{had}\dfrac{8x-16}{x^2-4}$

(c) $\underset{x\to 0}{had}\dfrac{2x^2+7}{\sqrt{3x^4+4}}$

[8 markah]

2. Given

$$f(x)=\begin{cases}2 & , \quad x\le -2\\ ax+b & , \quad -2<x<4,\\ -2 & , \quad x\ge 4\end{cases}$$

find the values of *a* and *b* if *f* is continuous everywhere.

[7 marks]

2. Diberi

$$f(x)=\begin{cases}2 & , \quad x\le -2\\ ax+b & , \quad -2<x<4,\\ -2 & , \quad x\ge 4\end{cases}$$

dapatkan nilai-nilai a dan b jika f selanjar di mana-mana.

[7 markah]

…3/-

3. (a) Find $\frac{dy}{dx}$ of the following functions:

(i) $y = e^{x^2} \sin 3x$

(ii) $x^2 + xy + y^2 = 7$

(iii) $y = (1 + \tan x)^{2x}$

[12 marks]

(b) A cylindrical swimming pool of height *h* is being filled with water from a hose at a rate of 1.5 cubic meters per second. How fast is the water level increasing if its radius is 6 meters?

[6 marks]

3. (a) Dapatkan $\frac{dy}{dx}$ bagi fungsi-fungsi berikut:

(i) $y = e^{x^2} \sin 3x$

(ii) $x^2 + xy + y^2 = 7$

(iii) $y = (1 + \tan x)^{2x}$

[12 markah]

(b) Suatu kolam mandi berbentuk selinder dengan ketinggian h dipenuhi air dari suatu hos pada kadar 1.5 kubik meter sesaat. Berapakah kadar paras air meningkat jika jejarinya ialah 6 meter.

[6 markah]

4. (a) Consider the equation $x^3 + 2x - 4 = 0, \quad 1 \le x \le 2$.

(i) Show that by using Newton's formula, the solutions of the equation can be deduced to $x_{n+1} = \dfrac{2x_n^3 + 4}{3x_n^2 + 2}, \quad n = 1, 2, 3.....$

(ii) Hence, show that the approximated root is 1.17951 correct to 5 decimal places.

[9 marks]

(b) The photosynthesis rate P (mg carbon/m^3/h) of phytoplankton, is modelled by the function

$$P(I) = \frac{100I}{I^2 + I + 4}$$

where $I > 0$ is the light intensity and measured in thousands of foot-candles. Find the light intensity when P is maximum.

[8 marks]

4. (a) Pertimbangkan suatu persamaan $x^3 + 2x - 4 = 0, \quad 1 \le x \le 2.$

(i) Tunjukkan bahawa dengan menggunakan rumus Newton, penyelesaian persamaan ini boleh dideduksikan kepada

$$x_{n+1} = \frac{2x_n^3 + 4}{3x_n^2 + 2}, \; n = 1, 2, 3.....$$

(ii) Seterusnya, tunjukkan bahawa punca hampiran ialah 1.17951 tepat kepada 5 tempat perpuluhan.

[9 markah]

(b) Kadar fotosintesis P (mg carbon/m^3/h) plytoplankton, dimodelkan oleh fungsi

$$P(I) = \frac{100I}{I^2 + I + 4}$$

dengan $I > 0$ ialah keamatan cahaya dan diukur dalam beribu kaki-lilin. Dapatkankan keamatan cahaya apabila P adalah maksimum.

[8 markah]

5. Given $f(x) = \dfrac{2x}{x^2 - 4}$, find

(a) all asymptotes of f.

(b) the interval on which f is increasing or decreasing.

(c) the local maximum and minimum point, if any.

(d) the interval of concavity and the inflection points, if exist.

Hence, sketch the graph of *f*.

[20 marks]

5. Diberi $f(x) = \dfrac{2x}{x^2 - 4}$, *dapatkan*

(a) kesemua asimptot untuk f.

(b) selang bagi f *menokok atau menyusut.*

(c) titik maksimum dan titik minimum tempatan, jika ada.

(d) selang kecekungan dan titik lengkuk balas, jika wujud.

Seterusnya, lakarkan graf f.

[20 markah]

6. Evaluate the integrals.

(a) $\displaystyle\int \frac{\cos x}{\sqrt{4 + 3\sin x}}\, dx$

(b) $\displaystyle\int \left(x^2 - 5x\right) e^x\, dx$

(c) $\displaystyle\int_0^1 \frac{x^3}{x^2 + 2x + 1}\, dx$

[20 marks]

6. Nilaikan kamiran.

(a) $\int \frac{\cos x}{\sqrt{4+3\sin x}}\,dx$

(b) $\int \left(x^2-5x\right)e^x\,dx$

(c) $\int_0^1 \frac{x^3}{x^2+2x+1}\,dx$

[20 markah]

7. (a) Sketch the region bounded by $y=2x-x^2$ and $y=x$.
Hence, set up, but do not evaluate the integral for

(i) the area of the region.

(ii) the volume of the solid obtained by rotating the region about the line $x=1$.

(b) Set up, but do not evaluate the integral for the length of the curve $y=\int_0^x \sqrt{\cos 2t}\ dt$ from $x=\pi/4$ to $x=1$.

[10 marks]

7. (a) Lakarkan rantau yang dibatasi oleh $y=2x-x^2$ dan $y=x$.
Seterusnya, nyatakan kamiran, tanpa menilaikannya untuk

(i) luas rantau.

(ii) isipadu pepejal yang dihasilkan dengan memutar rantau sekitar garis $x=1$.

(b) Nyatakan kamiran, tanpa menilaikannya untuk panjang lengkung $y=\int_0^x \sqrt{\cos 2t}\ dt$ dari $x=\pi/4$ ke $x=1$.

[10 markah]